Linear Algebra Assignment #5

(Not to be collected.)

1) [Eigenvalues and eigenvectors] Find the eigenvalues and a set of linearly independent eigenvectors of the matrices A, B, and P.
   a) A=$\begin{bmatrix} 5 & 5 \\ 6 & 4 \end{bmatrix}$
   b) B=$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
   c) P=$\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$

2) [Eigenvalues and eigenvectors proofs] Prove the following claims.
   a) For a projection matrix P, eigenvalues are either 0 or 1.
   b) Let A be an n by n matrix. Let x_1 , x_2 ,..., x_n be the eigenvectors of A with distinct eigenvalues λ_1, λ_2, ...,λ_n respectively. Then the eigenvectors x_i $(i = 1, \dots, n)$ are linearly independent.
   c) Diagonalizable matrices A, B share the same eigenvectors matrix S if and only if AB=BA.
   d) A matrix A has an inverse matrix A^{-1} if and only if it does not have zero as an eigenvalue. If nonzero λ_i are the eigenvalues of A, then the eigenvalues of A^{-1} are $1/\lambda_i$.

3) [Powers of a matrix] Let A be the matrix which has an eigenvector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ corresponding to the eigenvalue 3 and has an eigenvector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ corresponding to the eigenvalue -2 and also let v= $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, find A^{2000}v.

4) [Diagonalization] Find the eigenvalues of A and if possible diagonalize A. $A = S\Lambda S^{-1}$
   a) A=$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
   b) A=$\begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$

5) [SVD] Find the singular value decomposition of the matrices A, B, and S. Express B and S as $\sigma u v^T$ after calculating two pieces with SVD.

$$A=\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix},\ B=\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 3 \end{bmatrix},\ S=\begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 2 & 1 & 1 \end{bmatrix}$$

6) [Complex Numbers]
Let z1=-2+2i, and z2=1+$\sqrt{3}$i
Find $z1z2$, z1/z2 in polar form.

7) [Complex Matrices]
Let F_4 be a 4 by 4 Fourier matrix, and v be a vector

$$F_4 = \begin{bmatrix} i^0 & i^0 & i^0 & i^0 \\ i^0 & i & i^2 & i^3 \\ i^0 & i^2 & i^4 & i^6 \\ i^0 & i^3 & i^6 & i^9 \end{bmatrix},\ v=\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

a) Calculate $x = F_4 v$.
b) Find the unitary matrix kF_4 where k is a number.

8) [Linear Transformations]
The parts a, b, and c are seperate questions.
a) Let T:$R^3 \rightarrow P_2[x]$ be a linear transformation with T([x, 0, 0])=x+1, T([0, x, 0])=x^2-x, and T([0, 0, x])= x^2, find T([a, b, c]), also find the standard matrix A for the transformation.
b) Show that the Laplace transform is a linear operator.
c) $Q = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$. Let M be any 2 by 2 matrix and T be T(M)=QM. Is T a linear transformation? Is T one-to-one? Is T onto?

9) [Differential Equations] Solve the following the system of equations.
a) x'=Ax : $A = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$
b) x'=Ax : $A = \begin{bmatrix} 1 & -4 \\ 4 & -7 \end{bmatrix}$, x(0)=$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$